Mean p_t fluctuations from 2- and 4-particle correlations

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Recent results on event-by-event mean transverse momentum, $\langle p_t \rangle$, fluctuations in ultra-relativistic heavy ion collisions are briefly reviewed. We conclude that the observed fluctuations are in a rough agreement with that expected for the independent superposition of nucleon-nucleon collisions. We further discuss the possibility of the use of the forth order cumulants of the particle transverse momentum distribution in order to access the fluctuations related to collective phenomena.

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One of the most interesting questions in the field of the ultra-relativistic heavy ion collisions is the question of the hadronization of the system. Is the system thermalized/equilibrated? Does the system evolution include the phase transition? Event-by-event fluctuations, e.g. of the mean transverse momentum, are considered to be one of the important tools to answer these questions [1]. The fluctuations depend on the nature of the phase transition. A first order phase transition may lead to large fluctuations in energy density due to formation of QGP droplets[2, 3]. Second order phase transitions may lead to divergence in the specific heat; it would also increase the fluctuations in energy density due to long range correlations in the system [4]. One could observe them as fluctuations in mean transverse momentum if matter freezes out at the critical temperature T_c [4, 5, 6, 7].

The centrality dependence of the fluctuations is an important observable. If the fluctuations are due to the particle production via some kind of clusters (e.g., resonances, strings, (mini)jets, independent NN-collisions, etc.) and the relative production of such clusters do not change with centrality, the magnitude of the dynamical part in fluctuations should be inversely proportional to the number of the clusters, therefore to the particle multiplicity. New physics should appear as a deviation from such a dependence. There could be two reasons for the change in the centrality dependence. First, such collective phenomena as phase transition affect many particles in the system (unlike as in the scenario of particle production via a few particle clusters), and, therefore have different multiplicity (centrality) dependence. Second, new phenomena are expected to happen at some critical particle density, which in turn depends on centrality.

Consider fluctuations in the inverse slope of $m_t = \sqrt{m^2+p_t^2}$ distribution (effective temperature fluctuations). Such fluctuations, for example, could be due to event-by-event fluctuations in radial expansion velocity. Then, depending on the slope, on average, the transverse momenta of all particles would be lower or higher compared to the average over all events. It results in (positive) correlations between transverse momenta of different particles [8, 9]. One can quantify such correlations by the particle transverse momenta covariance

$$\kappa_{2,p_t} \equiv \mathrm{cov}(p_{t,i}, p_{t,j}) = \langle \delta p_{ti} \delta p_{tj} \rangle_{i \neq j} = \sigma^2_{\langle p_t \rangle, dynam}, \quad (1)$$

where $\delta p_{ti} = p_{t,i} - \overline{p_t}$, with $\overline{p_t}$ being the inclusive mean transverse momentum. It can be also useful to consider dimensionless quantities like $\sigma^2_{\langle p_t \rangle, dynam}/\sigma^2_{p_t, inclusive}$ or $\sigma^2_{\langle p_t \rangle, dynam}/\overline{p_t}^2$. The notation, $\sigma^2_{\langle p_t \rangle, dynam}$ [8], comes from the fact that this quantity equals to the difference between the actual variance of the event mean p_t distribution and the expected width due to the statistical fluctuations[19] in $\langle p_t \rangle$. The latter is due to the finite event multiplicity.

$$\begin{aligned} \sigma^2_{\langle p_t \rangle} &= \frac{1}{M^2} (\sum_i \delta p_{ti})^2 \\ &\approx \sigma^2_{\langle p_t \rangle, stat} + \frac{M-1}{M} \sigma^2_{\langle p_t \rangle, dynam}, \end{aligned} \quad (2)$$

where

$$\sigma^2_{\langle p_t \rangle, stat} = \frac{\sigma^2_{p_t, inclusive}}{M}, \quad (3)$$

and M is the multiplicity.

Event-by-event dynamical fluctuations have also been analyzed by several experiments using the so called Φ_{p_t} [10] measure (the approximate relation to $\sigma^2_{\langle p_t \rangle, dynam}$ is taken from [8]):

$$\begin{aligned} \Phi_{p_t} &\equiv \sqrt{\langle (\langle p_t \rangle - M\overline{p_t})^2 \rangle / \langle M \rangle} - \sigma_{p_t, inclusive} \\ &\approx \frac{\sigma^2_{\langle p_t \rangle, dynam} \langle M \rangle}{2\sigma_{p_t, inclusive}}. \end{aligned} \quad (4)$$

and very close to it the difference factor $\Delta\sigma_{p_t}$ [11]: $\Delta\sigma_{p_t} \equiv (\sqrt{\langle M \rangle} \sigma_{p_t, inclusive} - \sigma_{p_t, inclusive}) \approx \Phi_{p_t}$.

Using the above relations to compute κ_{2,p_t} whenever Φ_{p_t} being reported, we arrive to the conclusion that in *central* collisions of heavy nuclei such as gold and lead, relative fluctuations in mean transverse momentum, $\sigma_{\langle p_t \rangle, dynam}/\overline{p_t}$ is of the order of 1 – 1.5%. For the 6% most central collisions STAR reports [11] the preliminary results of $\sigma_{\langle p_t \rangle, dynam}/\overline{p_t} = 1.2 \pm 0.2$. The PHENIX measurements [12] have larger uncertainties, but if averaged over different centralities (assuming Φ_{p_t} does not change with centrality), this measurements yield for the central Au+Au collisions the value of $\sigma_{\langle p_t\rangle,dynam}/\overline{p_t} = 1.4 \pm 0.9$. In Pb+Pb and Pb+Au collisions at CERN SPS ($\sqrt{s_{NN}} = 17$ GeV) Φ_{p_t} have been measured by NA49 [13] and NA45/CERES [14]. The published NA49 result [13] $\Phi_{p_t} = 0.6 \pm 1$ MeV/c was obtained for the rapidity region $4.0 < y < 5.5$ and 5% most central collisions. The observed fluctuations are extremely small, but it would be incorrect to compare these numbers with RHIC measurements, since NA49 results were obtained in the forward rapidity region. The CERES collaboration at the SPS has measured the fluctuations in the central rapidity region. They report $\Phi_{p_t} = 7.8 \pm 0.9$ MeV/c in central Pb+Au collisions [14]. As seen from Eq. 4, Φ_{p_t} is directly proportional to the number of reconstructed tracks used for its calculation (subject to acceptance cuts and tracking efficiency), which complicates the comparison of the results from different experiments. For a rough comparison one can take into account the CERES multiplicity $\sim$130, $\langle\langle p_t\rangle\rangle \approx$420 GeV/c and $\sigma_{pt,\,inclusive} \approx 0.270$ GeV/c. Then $\Phi_{p_t} \approx 8$ MeV corresponds to $\sigma_{\langle p_t\rangle,dynam}/\overline{p_t} \approx 1.4\%$. The centrality dependence of $\langle p_t\rangle$ fluctuations, whenever studied, is consistent with particle production via clusters picture within about 30%.

The dynamical part of mean transverse momentum fluctuations has been measured at the ISR [15] in pp collisions. This was done by analyzing the multiplicity dependence of $\sigma_{\langle p_t\rangle}$ under the assumption that in pp collisions the dynamical part in $\langle p_t\rangle$ fluctuations does not depend on multiplicity. It was observed that $\sigma_{\langle p_t\rangle,dynam}/\langle\langle p_t\rangle\rangle \approx 12\%$. Rescaling of this quantity with (the square root of) the ratio of multiplicity densities in pp and $Au+Au$ collisions yields the fluctuations in $Au+Au$: $\sigma_{\langle p_t\rangle,dynam}/\langle\langle p_t\rangle\rangle \approx 0.8\%$, about 50% less than that observed in AA collisions. Rescaling the quantity with the number of participants gives even better agreement.

The non-zero value of κ_{2,p_t} could be due to different reasons: including such as resonance decays or jet production, which are not "interesting" from the point of view of the collective phenomena under search in nuclear collisions. In principle, selecting specific pairs for the averaging in Eq. 1 one can suppress or enhance the contribution to $\langle p_t\rangle$ fluctuations of different origin. For example, one can correlate: (1) particles in two disjoint pseudo-rapidity regions. The 'gap' between the two regions eliminates effects such as quantum statistics (Bose-Einstein and Fermi-Dirac) correlations or Coulomb final state interactions. (2) Positive particles with negative particles, which is expected to enhance the contribution from resonances. All those tricks do not solve the problem completely. Below we discuss the possibility of the use of 4-particle correlations (cumulants) in order to evaluate genuine collective phenomena contribution to mean p_t fluctuations:

$$\kappa_{4,p_t} = \langle \delta p_{t_i} \delta p_{t_j} \delta p_{t_k} \delta p_{t_m}\rangle - 3\langle \delta p_{t_i} \delta p_{t_j}\rangle^2 \tag{5}$$

with all i, j, k, and m being different. The relative contribution of the collective phenomena and particle production via clusters to the cumulant κ_{4,p_t} is enhanced by a factor of $\sim M_{total}/M_{cluster}$ compared to the relative contribution to κ_{2,p_t}. The notation $M_{cluster}$ is used here for the average cluster decay multiplicity, and M_{total} is the total event multiplicity.

In [16] cumulants have been proposed for the study of multiplicity fluctuations. Recently, four particle azimuthal correlations (cumulants) [17] have been successfully used by the STAR Collaboration in the analysis of the azimuthal correlations for the measurements of elliptic flow [18]. With modern statistics the technique works well even for relatively small signals (two particle correlations of the order of 10^{-4} and smaller. This is just the range of the correlation magnitude needed for the 4-particle correlations mean p_t analysis (recall that $\kappa_{2,p_t}/\langle p_t\rangle^2 \sim (1-1.5\%)^2$. In many particle cumulant mean p_t analysis one can use the generating functions similar to the ones used in [17, 18].

The contribution to κ_{4,p_t} from the inverse slope fluctuations to the first order is proportional to the corresponding cumulant of the inverse slope distribution. For the case of of an equal mixture of event with two different slopes T_1 and T_2, the corresponding cumulant $\kappa_{4,T} = -(\Delta T)^4/8$, ($\Delta T = T_1 - T_2$). Note that for some specific inverse slope distributions the cumulant could be small (for a rectangular distribution with width ΔT it is $\kappa_{4,T} = -(\Delta T)^4/120$, and for a Gaussian distribution it is zero). The very small values of the cumulants may complicate the analysis. The use of additional weights in Eq. 5 could somewhat help in this case.

In **summary**, we observe that while clear correlations in particle transverse momenta has been measured in heavy ion collisions at SPS and RHIC, no evidence that the correlations are due to collective phenomena have been found so far. The centrality dependence of the observed correlations, as well as comparison to pp collisions at similar energies, is roughly consistent with the picture of particle production via clusters. In order to disentangle the contribution to the mean p_t fluctuations coming from collective phenomena we propose to use many-particle correlations.

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